Scaling laws in the central region of confined turbulent thermal convection

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In confined turbulent thermal convection, the velocity is separated into two parts: one that is correlated with some function of the temperature fluctuations, and thus associated with the plume velocity, and the other part, the background velocity, which is uncorrelated with any function of the temperature fluctuations. As a result, one should focus on the plume velocity, and not the whole velocity, and the temperature when studying the scaling behavior. In this paper, a phenomenological theory for the scaling behavior in the central region of confined turbulent thermal convection is presented. The spatial (temporal) plume velocity structure functions are found to have the same scaling behavior as the spatial (temporal) temperature structure functions. For $\tau \geqslant \tau_b$, where the buoyant scale τ_b is determined in terms of measurable quantities, the scaling exponents of the temporal temperature structure functions and hence those of the temporal plume velocity structure functions are obtained. These results are checked against experimental measurements, and good agreement is found.

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I. INTRODUCTION

A key issue in turbulence research is to make sense of the complex fluctuations of velocity and other physical quantities of interest in a turbulent fluid flow. To study the statistics of, say, velocity fluctuations, it is common to study the nth-order velocity structure functions $\tilde{S}_n(r) \equiv \langle |v_r|^n \rangle$, which are moments of the velocity difference $v_r \equiv [\vec{v}(\vec{x}+\vec{r},t) - \vec{v}(\vec{x},t)] \cdot \vec{r}/r$ between two points separated by a vector $\vec{r}$. Here $\langle \cdots \rangle$ denotes an ensemble average and is usually evaluated as a long-time average in experiments and numerical calculations. In fully developed turbulent flows governed by the Navier-Stokes equations, it is generally believed that $\tilde{S}_n(r) \sim r^{\zeta_n}$, when r is within the inertial range, with the scaling exponents ζ_n equal to the Kolmogorov 1941 prediction of $n/3$ [1] plus corrections. The inertial range is the intermediate range of length scales that are smaller than those of the energy input and larger than those affected directly by molecular dissipation. As of today, one is not yet able to calculate the corrections from the Navier-Stokes equations, but a phenomenological model [2] gives values that are in good agreement with experimental data. On the other hand, there is not yet consistent understanding, even at a phenomenological level, of the scaling behavior in turbulent thermal convection confined in a closed box of fluid heated from below and cooled on the top, a system of much research interest (see, e.g., [3–5]).

In confined turbulent thermal convection, the dynamics is driven by buoyancy, resulted from an applied temperature difference Δ across the height L of the box. In the Boussinesq approximation, the governing equations are [6]

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = -\vec{\nabla} p + \nu \nabla^2 \vec{v} + \alpha g (T - T_0) \hat{z}, \tag{1}$$

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \vec{\nabla} T = \kappa \nabla^2 T, \tag{2}$$

$$\vec{\nabla} \cdot \vec{v} = 0. \tag{3}$$

Here, $\vec{v}$, T, and p are, respectively, the velocity, temperature, and pressure divided by the density, g is the acceleration due to gravity, T_0, α, ν, and κ are, respectively, the mean temperature, volume expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid, and $\hat{z}$ is the unit vector in the vertical direction. The statistics of temperature fluctuations are also of interest and one similarly studies the temperature structure functions

$$\tilde{R}_n(r) \equiv \langle |T_r|^n \rangle \tag{4}$$

with

$$T_r \equiv T(\vec{x}+\vec{r},t) - T(\vec{x},t). \tag{5}$$

The state of fluid flow is characterized by the geometry of the box and two dimensionless parameters, the Rayleigh number $\text{Ra} = \alpha g \Delta L^3/(\nu\kappa)$ and Prandtl number $\text{Pr} = \nu/\kappa$. An interesting question is how the presence of a buoyant force affects the scaling properties of turbulence. Specifically, one would like to understand the scaling behavior of $\tilde{S}_n(r)$ and $\tilde{R}_n(r)$ in confined turbulent thermal convection.

There have been various experimental and numerical studies of the scaling behavior of confined turbulent thermal convection. Experiments using low-temperature helium gas first revealed [7] that the temperature frequency power spectrum measured in the central region obeys a scaling law of $f^{-1.4}$. If one relates the frequency f to the wave number k by $f = 2\pi k U$ for some velocity U, then this observation resembles the scaling results of Bolgiano [8] and Obukhov [9]. In their original work for stably stratified turbulence (see [10] for a review), the vertical velocity and temperature power spectra were predicted to have a scaling behavior of $k^{-11/5}$ and $k^{-7/5}$, respectively, when k is small enough, based on dimensional considerations. The power spectra are Fourier transforms of the second-order structure functions up to some constants. Extending their dimensional considerations to structure functions of all orders, Bolgiano-Obukhov (BO) scaling then reads

$$\tilde{S}_n^V(r) \equiv \langle |v_{zr}|^n \rangle \sim r^{3n/5}, \quad \tilde{R}_n(r) \sim r^{n/5}, \tag{6}$$

where $v_{zr} \equiv v_z(\vec{x}+\vec{r},t) - v_z(\vec{x},t)$ and the superscript V stands for vertical. Some theoretical work [11–14] argued that Eq. (6) should also hold in confined turbulent thermal convection but some [15,16] argued otherwise. There have been further experimental reports that the temperature and vertical velocity frequency power spectra have a scaling behavior of $f^{-1.4}$ and $f^{-2.2}$, resembling the BO scaling [17–19]. However, these spectra were measured at different locations with the temperature frequency spectrum measured at the center and the velocity spectrum measured near the bottom plate and sidewall of the convection cell. Moreover, evidence against various relations between vertical velocity and temperature implied by BO scaling was reported [20,21]. Results from numerical studies are just as confusing. Simulations using the lattice Boltzmann scheme [22–24] with periodic horizontal boundary conditions seem to favor BO scaling. However, direct numerical simulations of Eqs. (1)–(3) with physical boundary conditions [25,26] revealed results that are in disagreement with BO scaling.

Flow visualizations reveal that organized fluid motion exists in confined turbulent thermal convection. These coherent flow structures include plumes, which are flow structures generated from the thermal boundary layers by buoyancy, and a large-scale mean circulation spanning the whole box, commonly known as the wind [27]. Recently, we have developed a scheme [28] to extract information about the plumes from simultaneous velocity and temperature measurements. In this scheme, it was argued that the velocity at any point is the sum of the plume velocity and the background velocity; the background velocity is uncorrelated with any function of the temperature correlations and only the plume velocity is correlated with some function of the temperature fluctuations. This work thus suggests that one should focus on the plume velocity and not the whole velocity when studying the possible effects of buoyancy on statistical properties of turbulence. In other words, one should focus on the plume velocity and temperature structure functions when studying the scaling behavior of confined turbulent thermal convection.

In this paper, we follow this idea and present scaling laws in the central region of confined turbulent thermal convection. We show that the plume velocity fluctuations have the *same* scaling behavior as the temperature fluctuations. This feature is in contrast with the BO scaling for vertical velocity and temperature fluctuations. Scaling exponents for the temperature structure functions and thus the plume velocity structure functions are derived. Our results are checked against experimental measurements, and good agreement is found.

II. THEORY

A. Relation between temperature and plume velocity structure functions

For completeness, we first review the main ideas of the scheme developed earlier [28] to extract the velocity of the plumes. At any point, the velocity $\vec{v}(t)$ is separated into two parts:

$$\vec{v}(t) = \vec{v}_p(t) + \vec{v}_b(t). \tag{7}$$

One part is the background velocity, denoted as $\vec{v}_b(t)$, which should be uncorrelated with any function of temperature fluctuations. This decomposition is achieved by taking $\vec{v}_p$ as the conditional average of $\vec{v}$ for a given value of the temperature fluctuations at the same time t:

$$\vec{v}_p(t) = \langle \vec{v} | T(t) \rangle = \int \vec{v} \mathcal{P}(\vec{v}|T) d\vec{v} \tag{8}$$

where $\mathcal{P}(\vec{v}|T)$ is the conditional probability density function (PDF) of $\vec{v}$ on T. In practice, this conditional average can be calculated from simultaneous measurements of velocity and temperature as follows. At each time t, find the times t' at which the temperature measurements $T(t')$ have values falling within $T(t) \pm \delta$. Then the average of those velocity measurements $\vec{v}(t')$ at such times t' is $\langle \vec{v} | T(t) \rangle$. In flows where the conditional average $\langle \vec{v} | T(t) \rangle$ equals the usual average $\langle \vec{v} \rangle$, buoyant structures are not present. Thus we consider only the case $\langle \vec{v} | T(t) \rangle \neq \langle \vec{v} \rangle$ such that $\langle \vec{v} | T(t) \rangle$ is a function of T and thus t.

We now show that $\vec{v}_b$ so defined would indeed be uncorrelated with any function of temperature fluctuations, as claimed. Using Eq. (7) with Eq. (8), we get

$$\langle \vec{v} f(T) \rangle = \langle \langle \vec{v} | T(t) \rangle f(T) \rangle + \langle \vec{v}_b f(T) \rangle \tag{9}$$

for any function $f(T)$ of T. Note that

$$\begin{aligned} \langle \langle \vec{v} | T(t) \rangle f(T) \rangle &= \int \left(\int \vec{v} \mathcal{P}(\vec{v}|T) d\vec{v} \right) f(T) \mathcal{P}(T) dT \\ &= \int \int \mathcal{P}(\vec{v},T) \vec{v} f(T) d\vec{v}\, dT = \langle \vec{v} f(T) \rangle \end{aligned} \tag{10}$$

where $\mathcal{P}(\vec{v},T)$ is the joint PDF of $\vec{v}$ and T. In getting Eq. (10), we have made use of the result

$$\mathcal{P}(\vec{v}|T)\mathcal{P}(T) = \mathcal{P}(\vec{v},T). \tag{11}$$

Hence Eqs. (9) and (10) imply that

$$\langle \vec{v}_b(t) f(T) \rangle = 0 \tag{12}$$

for any function $f(T)$, proving that $\vec{v}_b(t)$ is uncorrelated with any function of T and averages to zero over time. This justifies the interpretation of $\vec{v}_b(t)$ as the background velocity fluctuation.

The remaining part $\vec{v}_p$ in $\vec{v}$ would be correlated with some function, albeit unknown *a priori*, of the temperature fluctuations. Plumes are flow structures generated by buoyancy so their velocity should be related to the temperature fluctuations in some way. Hence $\vec{v}_p$ is naturally associated with the velocity of the plumes. As the buoyant force acts in the vertical direction, one expects

$$\vec{v}_p(t) \approx v_{pz}(t)\hat{z}. \tag{13}$$

In the central region, the mean velocity is small so the buoyant force is balanced by the viscous force. Since plumes are generated from the thermal boundary layers and remain as detached structures in the central region, they have the length scale of the thermal boundary layer thickness λ_{th}. Hence dimensional analysis gives

$$\alpha g(T-T_0) \approx a\frac{\nu v_{pz}}{\lambda_{th}^2} \tag{14}$$

where a is a numerical factor of order 1. This velocity decomposition was carried out and Eqs. (13) and (14) were confirmed with $a \approx 1/4$ [28].

With the result of Eq. (14), we can now readily relate the plume velocity and temperature structure functions. Specifically, Eq. (14) implies that

$$v_{pzr} \approx \frac{4\alpha g \lambda_{th}^2}{\nu} T_r \equiv c T_r, \tag{15}$$

$$v_{pz\tau} \approx \frac{4\alpha g \lambda_{th}^2}{\nu} T_\tau \equiv c T_\tau, \tag{16}$$

where

$$v_{pzr} \equiv v_{pz}(\vec{x}+\vec{r},t) - v_{pz}(\vec{x},t), \tag{17}$$

$$v_{pz\tau} \equiv v_{pz}(\vec{x},t+\tau) - v_{pz}(\vec{x},t), \tag{18}$$

$$T_\tau \equiv T(\vec{x},t+\tau) - T(\vec{x},t). \tag{19}$$

Define the spatial and temporal plume velocity structure functions, respectively, as

$$\tilde{P}_n(r) \equiv \langle |v_{pzr}|^n \rangle, \tag{20}$$

$$P_n(\tau) \equiv \langle |v_{pz\tau}|^n \rangle, \tag{21}$$

and the temporal temperature structure functions as

$$R_n(\tau) \equiv \langle |T_\tau|^n \rangle. \tag{22}$$

Then we have the interesting result that, in the central region, the plume velocity and temperature structure functions, both spatial and temporal, are proportional to one another and hence have the same scaling behavior:

$$\tilde{P}_n(r) \approx c^n \tilde{R}_n(r), \tag{23}$$

$$P_n(\tau) \approx c^n R_n(\tau). \tag{24}$$

This feature is in contrast with the BO scaling for vertical velocity and temperature fluctuations [see Eq. (6)]. We shall return to this point later.

B. Scaling exponents of temporal temperature and plume velocity structure functions

We shall proceed to obtain the scaling behavior of the temperature structure functions and thus that of the plume velocity structure functions. Since temporal structure functions are commonly studied because spatial differences between two points, separated by varying r, are difficult to measure in experiments, we shall focus on temporal structure functions.

The statistics of the temporal temperature difference T_τ in the central region were studied and found to be intermittent in that the functional form of the PDFs of T_τ changes with τ [29]. The conditional PDFs of T_τ at fixed values of the local thermal dissipation rate were studied and found [30] to become Gaussian for τ sufficiently large. This result indicates that, for sufficiently large τ, the intermittency of T_τ can be solely attributed to the variations of the local thermal dissipation rate.

Based on these results, we propose that, for τ larger than some buoyant scale τ_b, the statistics of T_τ should depend only on $u_{rms}\tau$, χ_τ, and αg. Here, χ_τ is defined as the thermal dissipation rate locally averaged over a time interval τ,

$$\chi_\tau(t) = \frac{1}{\tau}\int_t^{t+\tau} \kappa(\nabla T)^2 dt', \tag{25}$$

and u_{rms} is the rms velocity fluctuation at the center of the cell. Our use of χ_τ instead of the global mean thermal dissipation rate χ, an extension of Kolmogorov's refined similarity ideas [31] to turbulent convection. Dimensional analysis then gives

$$T_\tau \sim (\alpha g)^{-1/5} \chi_\tau^{2/5} (u_{rms}\tau)^{1/5}. \tag{26}$$

As a result, we obtain

$$R_n(\tau) \sim (\alpha g)^{-n/5} \langle \chi_\tau^{2n/5} \rangle (u_{rms}\tau)^{n/5} \sim \tau^{\xi_n} \tag{27}$$

where ξ_n are the scaling exponents of the temporal temperature structure functions, which are also the scaling exponents of the temporal plume velocity scaling exponents. Let

$$\langle \chi_\tau^n \rangle \sim \tau^{\mu_n}; \tag{28}$$

then we have

$$\xi_n = \frac{n}{5} + \mu_{2n/5}. \tag{29}$$

Equation (29) states that ξ_n are the BO values of $n/5$ plus corrections $\mu_{2n/5}$ due to variations of χ_τ. By assuming that the moments $\langle \chi_\tau^n \rangle$ satisfy a hierarchical structure of the She-Leveque form [2] and with some physical agruments, Ching and Kwok [32] derived the result that

$$\mu_n = 1 - \left(\frac{1}{3}\right)^n - \frac{2}{3}n. \tag{30}$$

Putting Eq. (30) into Eq. (29), we obtain an explicit expression for the scaling exponents of temporal temperature and temporal plume velocity structure functions:

$$\xi_n = 1 - \left(\frac{1}{3}\right)^{2n/5} - \frac{n}{15}. \tag{31}$$

Note that ξ_n becomes negative for sufficiently large n but this is not problematic. The usual arguments for positive exponents are based on the boundedness of the temperature field plus the scaling behavior holding for $\tau \to 0$. But here $R_n(\tau)$ in the central region scales as τ^{ξ_n} only for $\tau \geqslant \tau_b$, and τ_b remains finite even as Ra increases, as we shall show next.

At scale τ, the average power injected into the flow due to the buoyant forces is

$$\mathcal{P}_b(\tau) = \langle \alpha g T_\tau v_{z\tau} \rangle \approx \alpha g \langle T_\tau v_{pzr} \rangle. \quad (32)$$

Using Eq. (16), $\mathcal{P}_b(r) \approx \alpha g c \langle T_\tau^2 \rangle$ and increases with τ. We expect buoyant forces to be significant when $\mathcal{P}_b(\tau)$ exceeds the mean energy dissipation rate ϵ, or equivalently when $\tau \geqslant \tau_b$, where τ_b is given by

$$\mathcal{P}_b(\tau_b) = \epsilon. \quad (33)$$

Substituting Eqs. (16) and (26) into Eq. (33) gives

$$4(\alpha g)^{8/5} \lambda_{th}^2 \langle \chi_{\tau_b}^{4/5} \rangle (u_{rms} \tau_b)^{2/5} = \nu \epsilon. \quad (34)$$

From Eq. (28), we get

$$\frac{\langle \chi_{\tau_b}^{4/5} \rangle}{\chi^{4/5}} = \left(\frac{\tau_b}{\tau_L} \right)^{\mu_{4/5}} = \left(\frac{u_{rms} \tau_b}{L} \right)^{\mu_{4/5}} \quad (35)$$

where we have used the approximation $\chi_{\tau_L} \approx \chi$ with $\tau_L \equiv L/u_{rms}$ and χ being the mean thermal dissipation rate, averaged over the whole convection cell of volume V,

$$\chi \equiv \frac{1}{V} \int_{\text{whole cell}} \kappa (\nabla T)^2 d^3 r. \quad (36)$$

Putting these results together and using $L/(2\lambda_{th}) \approx \mathrm{Nu}$, where the Nusselt number Nu is the dimensionless heat flux, and the exact results [15] of $\epsilon = (\nu \kappa^2 / L^4) \mathrm{Nu}\,\mathrm{Ra}$ and $\chi = \kappa (\Delta / L)^2 \mathrm{Nu}$, we can express τ_b in terms of measurable quantities:

$$\left(\frac{u_{rms} \tau_b}{L} \right)^{\xi_2} \approx \mathrm{Ra}^{-3/5} \mathrm{Pr}^{2/5} \mathrm{Nu}^{11/5}. \quad (37)$$

Experiments show that Nu can be approximated as an effective power law of Ra with an exponent ranging from 2/7 to 1/3 [33]; hence the left-hand side of Eq. (37) scales as a positive exponent of Ra such that τ_b increases with Ra. Hence we have the intriguing result that the temperature fluctuations in the central region would become passivelike for sufficiently large Ra [34]. Furthermore, τ_b would always remain finite, as discussed earlier.

III. CHECKING RESULTS AGAINST EXPERIMENTAL MEASUREMENTS

In this section, we check our results against experimental measurements. Two sets of experimental data have been analyzed: one consists of temperature measurements taken at the center of a convection cell filled with low-temperature helium gas [35]; and the other of simultaneous temperature and velocity measurements taken at the center of a convection cell filled with water [36]. We refer to these two sets of experimental data as helium data and water data, respectively. From these two sets of data, we can study only temporal structure functions. Moreover, the plume velocity can be evaluated only for the water data.

We first check Eq. (24). Using the water data with $\mathrm{Ra} = 4.8 \times 10^9$, we calculate $P_n(\tau)$ and $R_n(\tau)$ and compare them. The results are shown in Figs. 1 and 2.

The proportionality of $P_n(\tau)$ with $R_n(\tau)$ is clearly seen. In Figs. 1 and 2, we also show the least-squares fit of $P_n(\tau) = a_n R_n(\tau)$ for $n=1$ and 2. The fitted values are $a_1 = 0.87 \pm 0.07$ and $a_2 = 0.70 \pm 0.05$. Thus, $a_2 = a_1^2$ within 10%, in agreement with Eq. (24).

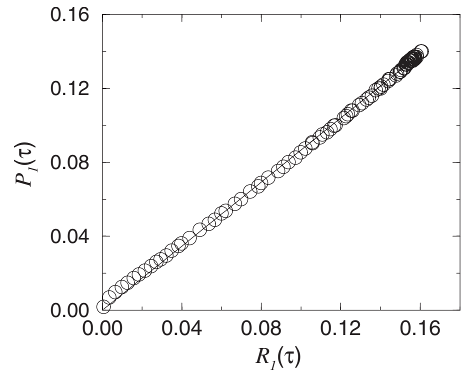

FIG. 1. Comparison of the temporal plume velocity structure functions $P_n(\tau)$ and temporal temperature structure functions $R_n(\tau)$ for $n=1$ using the water data with $\mathrm{Ra}=4.8 \times 10^9$. The plume velocity is in cm/s while the temperature is in degrees Celsius. The solid line is the least-squares fit of $P_1(\tau) = a_1 R_1(\tau)$ with the fitted value of $a_1 = 0.87 \pm 0.07$.

As discussed, the temporal temperature structure functions $R_n(\tau)$ and the moments of χ_τ were studied before using the helium data. In the evaluation of χ_τ, ∇T was approximated to be proportional to the temperature time derivative dT/dt, μ_n were measured and Eq. (30) was verified [32]. For the temporal temperature structure functions, scaling behavior was seen only when $R_n(\tau)$ was plotted against $R_2(\tau)$ [20], and the relative scaling exponents $\alpha_n = \xi_n / \xi_2$ in the large-τ regime were measured. Next, we use these reported results to check Eqs. (29) and (31). From Eqs. (29) and (31), we obtain

$$\alpha_n = \frac{n/5 + \mu_{2n/5}}{2/5 + \mu_{4/5}}, \quad (38)$$

$$\alpha_n = \frac{1 - (1/3)^{2n/5} - n/15}{1 - (1/3)^{4/5} - 2/15}. \quad (39)$$

We compare these theoretical predictions with the measured values of α_n for the data set with $\mathrm{Ra} = 7.3 \times 10^{10}$ that displays

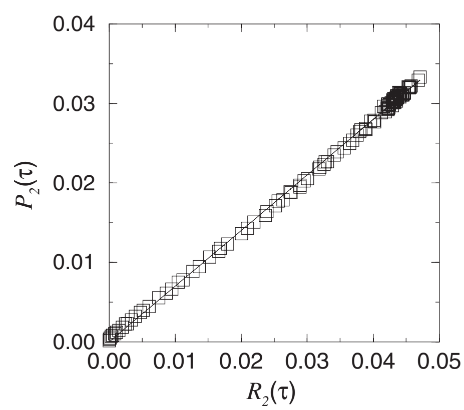

FIG. 2. Same as Fig. 1 for $n=2$. The fitted value of $a_2 = 0.70 \pm 0.05$.

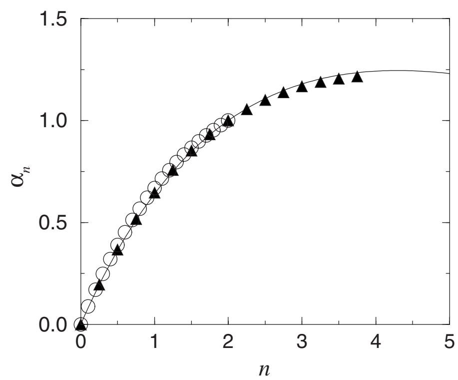

FIG. 3. Comparison of the measured values of the relative exponents α_n (circles) at the cell center with their theoretical values given by Eq. (38) with the measured values of μ_n (triangles), and with Eq. (39) (solid line).

the longest scaling range. Our results are shown in Fig. 3 and good agreement is seen. This data set at $\mathrm{Ra}=7.3\times 10^{10}$ contains 614 400 temperature measurements. Because of the relatively small number of measurements, α_n and μ_n can be accurately evaluated only for n up to around 2.

IV. PLUMES AND THE INVALIDITY OF BOLGIANO-OBUKHOV SCALING

According to BO scaling, the whole vertical velocity is correlated with some function of temperature, and the vertical velocity and the temperature structure functions have different scaling exponents. This is in contrast to what we find here. In this section, we shall understand further why BO scaling does not hold in confined turbulent thermal convection.

If one assumes that the statistics of the plume velocity difference $v_{pz\tau}$ depend only on $u_{rms}\tau$, χ_τ, and αg as in Eq. (26) for T_τ, then one gets the scaling exponents of the plume velocity as the BO values of $3n/5$ plus corrections. Thus the fact that the scaling exponents of the temporal plume velocity structure functions are the same as those of the temporal temperature structure functions and not the BO values plus corrections shows that the statistics of $v_{pz\tau}$ do not depend only on $u_{rms}\tau$, χ_τ, and αg. This is indeed clearly demonstrated in Eq. (16), in which there is an explicit reference to the length scale λ_{th}. This length scale appears because of the presence of the buoyant structures, plumes, which are formed from the thermal boundary layers and naturally have a length scale of the order of the thermal boundary layer thickness λ_{th}. Equation (16) also expresses an exchange of buoyant potential energy and the kinetic energy of the plumes. Hence, the presence of plumes introduces an additional length scale of λ_{th} for the exchange of buoyant potential energy and kinetic energy, and, as a result, ruins the original dimensional considerations that led to BO scaling. This also suggests that in the absence of buoyant structures, the scaling behavior of the vertical velocity and temperature fluctuations in turbulent thermal convection would be given by BO scaling plus intermittent corrections.

Now the confinement with the resulting boundary conditions is crucial to the formation of buoyant flow structures such as plumes. It is, therefore, not surprising that numerical simulations using different boundary conditions can lead to different scaling behavior. In particular, in simulations using periodic boundary conditions, one expects that thermal boundary layers will not be formed as effectively, and there are fewer or no plumes. This might explain why different scaling behavior was reported in different numerical studies using different boundary conditions [22–26]. Moreover, when the vertical velocity is dominated by the plume velocity, then the vertical velocity structure functions will have scaling behavior approximated by that of the plume velocity structure functions. In this case, the vertical velocity structure functions will have approximately the same scaling behavior as the temperature structure functions. This might be the case in the numerical study [26] in which the temporal vertical velocity and temperature spectra were found to have similar scaling.

Furthermore, in shell models, which are dynamical models modeling the cascade processes in turbulence (see, e.g., [37] for a review), there are no boundaries. As a result, shell models of turbulent thermal convection will be, by construction, free of plumes. Hence, the above arguments predict that, in a shell model of turbulent thermal convection, BO scaling plus intermittent corrections should be valid. This is indeed the case. Brandenburg constructed a shell model of turbulent thermal convection and reported BO scaling for the velocity and temperature spectra [38]. BO scaling plus intermittent corrections were also reported in a slightly modified model [39]. Based on our present work, we can derive the intermittency corrections and these results will be reported elsewhere.

V. SUMMARY AND CONCLUSIONS

In confined turbulent thermal convection, the velocity naturally separates into two parts: a part that is correlated with some function of the temperature fluctuations and thus associated with the plumes, and the background velocity, which is uncorrelated with any function of the temperature fluctuations. As a result, when studying scaling behavior of confined turbulent thermal convection, one should focus on the plume velocity and temperature structure functions.

Using a scheme that we developed earlier [28] to extract the velocity of plumes, we have derived a relation between the plume velocity and the temperature structure functions [Eqs. (23) and (24)]. We have found that the plume velocity structure functions, both spatial and temporal, are proportional to the temperature structure functions. Hence the plume velocity and temperature have the same scaling behavior. This result is supported by experimental measurements. It differs from BO scaling, in which the vertical velocity and the temperature have different scaling behavior. We have argued that the invalidity of the BO scaling is due to the presence of plumes in the problem. The plumes are formed from the thermal boundary layers and thus have a length scale of the thermal boundary layer thickness λ_{th}. As a result, the exchange of buoyant potential energy and the kinetic energy of the plumes involves explicitly this additional length scale λ_{th}. This ruins the original dimensional considerations that led to BO scaling, and hence BO scaling is invalid in confined turbulent thermal convection. Our work also suggests that, in the absence of buoyant flow structures like plumes, the scaling behavior will be given by BO scaling plus corrections. This suggestion is supported in the Brandenburg shell model of turbulent thermal convection.

We have proposed that for $\tau \geq \tau_b$, where buoyancy is significant, the statistics of the temporal temperature difference T_τ should depend only on $u_{rms}\tau$, χ_τ, and αg. Then using a hierarchical structure model [2] for the moments of χ_τ [32], we have derived the scaling exponents ξ_n of the temporal temperature structure functions and thus the temporal plume velocity structure functions. Our results are summarized in Eqs. (29) and (31), and checked to be in good agreement with experimental measurements. Moroever, the buoyant scale τ_b is determined in terms of measurable quantities as shown in Eq. (37), and is shown to increase with Ra.

In the present work, we have studied only the scaling behavior in the central region of the convection cell. It is known that confined turbulent thermal convection is inhomogeneous and different regions might have different scaling behavior. In particular, we propose that Eq. (26) is likely to hold also in other regions of the convection cell but that the statistics of χ_τ might be different in different regions. It would be interesting to check this. Furthermore, we have focused on temporal structure functions as they are commonly studied in experiments. Spatial velocity and temperature structure functions were recently measured in confined turbulent thermal convection. In the central region, the spatial temperature structure functions were found to have scaling exponents close to that of a passive scalar [40]. One possibility is that the measurements were taken at scales smaller than the buoyant scale so that buoyancy was not significant, and so the temperature is acting like a passive scalar. If this is not the case, then these new experimental results [40] indicate that the spatial and temporal temperature structure functions in the central region have different scaling behavior. Indeed, in the same study [40], the spatial and temporal vertical velocity structure functions near the sidewall were reported to have different scaling behavior. This indicated difference between spatial and temporal structure functions is interesting but needs to be demonstrated explicitly and studied in detail.

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